Nuclear Reactor Theory

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Introduction

- In a critical nuclear reactors, there is balance between number of produced and lost neutrons
  - neutrons are produced by fission
  - neutrons are lost by either absorption or escape
- Central problem in nuclear reactor designing is determination of its dimensions and composition
- Techniques for calculation of size and material composition of a homogeneous nuclear reactor are introduced here
- Criticality calculations are usually carried out by a group diffusion method
- The first part will deal with one-group method suitable for fast reactors and to some extent even for thermal systems

Fast Homogeneous Reactor

- Fast critical reactor with homogeneous mixture of fuel and coolant is considered
- The reactor has only one region, no reflector, it is so-called *bare reactor*
- The reactor is described by one-group diffusion equation:

$$D\nabla^2\Phi - \Sigma_a\Phi + s = \frac{1}{v}\frac{\partial\Phi}{\partial t} \tag{7-1}$$

- This equation is time-dependent and power of the reactor might increase or decrease

Fission Source of Neutrons

- Fission neutrons are the source of neutrons (s) in a nuclear reactor
- If Σ_f is fission cross-section of the fuel and ν number of neutrons emitted per one fission, source s can be expressed as:

$$s = \nu \Sigma_f \Phi$$

- If fission source does not balance neutron absorption and leakage, then right-hand side of equation (7-1) is nonzero
- Parameter k can be used to adjust the source strength and to reach a steady state diffusion equation:

$$D\nabla^2\Phi - \Sigma_a\Phi + \frac{1}{k}\nu\Sigma_f\Phi = 0 \tag{7-2}$$

One-Group Reactor Equation

- Quantity *geometric buckling (B^2)* is defined as:

$$B^2 = \frac{1}{D}\left(\frac{\nu}{k}\Sigma_f - \Sigma_a\right) \tag{7-3}$$

- Then previous equation (7-2) can be rewritten in form of *one-group reactor equation*:

$$\nabla^2\Phi + B^2\Phi = 0 \tag{7-4}$$

- The formula for buckling (7-3) can be solved for the constant k:

$$k = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a} \tag{7-5}$$

Multiplication Factor

$$k = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a} = \frac{\nu\Sigma_f\Phi}{DB^2\Phi + \Sigma_a\Phi} = \frac{\nu\Sigma_f\Phi}{-D\nabla^2\Phi + \Sigma_a\Phi}$$

- Physical interpretation of the previous equation is following:
  - numerator is the number of neutrons born in fission in the current generation
  - denominator represents neutrons lost from the previous generation
  - since all neutrons must be absorbed or leak from the reactor, the denominator must be also equal to the number of neutrons born in the previous generation
- This is definition of *multiplication factor* for a finite reactor
- It can be also defined as a neutron birth rate divided by sum of neutron absorption and leakage rate

Multiplication Factor for Infinite Reactor

- The neutron source term can be rewritten with neutron absorption. Let Σ_{aF} be cross-section for neutron absorption in fuel, then:

$$s = \eta \Sigma_{aF} \Phi$$

- Quantity η is called *neutron reproduction factor* and means number of produced neutrons per a single neutron absorbed in the fuel

- It can be further adjusted to:

$$s = \eta \frac{\Sigma_{aF}}{\Sigma_a} \Sigma_a \Phi = \eta f \Sigma_a \Phi \text{ , where f is } \frac{\Sigma_{aF}}{\Sigma_a}$$

- Quantity f is called *fuel utilisation factor* and means a fraction of neutrons absorbed in the fuel from all neutrons absorbed in the reactor

Multiplication Factor for Infinite Reactor (2)

- There is no escape of neutrons in an infinite nuclear reactor, all neutrons are either absorbed in fuel or coolant
- In one generation, a certain number of neutrons is born related to ν, all these neutrons must be absorbed expressed as $\Sigma_a \Phi$
- Of these neutrons $f \Sigma_a \Phi$ are absorbed in fuel and this leads to production of $\eta f \Sigma_a \Phi$ neutrons
- All these neutrons must be again absorbed
- It means that absorption of $\Sigma_a \Phi$ neutrons in one generation leads to absorption of $\eta f \Sigma_a \Phi$ neutrons in the following generation

- Absorption of neutrons is directly related to production of neutrons, therefore *multiplication factor in an infinite fast reactor* is defined as:

$$k_\infty = \frac{\eta f \Sigma_a \Phi}{\Sigma_a \Phi} = \eta f \tag{7-6}$$

Buckling for Critical Reactor

- Value of η and f depend only on material composition and k_∞ is therefore identical for all infinite bare reactors with the same material composition
- The neutron source term can be written as:

$$s = k_\infty \Sigma_a \Phi$$

- And one-group reactor equation (7-2) can be transformed to:

$$DB^2\Phi + \Sigma_a\Phi - \frac{k_\infty}{k}\Sigma_a\Phi = 0$$

- If the reactor is just critical (k = 1) the right-hand side is zero and:

$$DB^2\Phi - (k_\infty - 1)\Sigma_a\Phi = 0$$

Buckling for Critical Reactor (2)

- Dividing by D and introducing one-group diffusion area $L^2 = D/\Sigma_a$ leads to:

$$B^2\Phi - \frac{k_\infty - 1}{L^2}\Phi = 0$$

- The above equation can be solved for geometric buckling factor of a critical reactor

$$B^2 = \frac{k_\infty - 1}{L^2}$$

- It will be further shown that buckling factor determines shape of neutron flux and sets a condition for a reactor to be critical

Slab Reactor

- First example is a critical infinite slab reactor with thickness a. The reactor equation (7-4) is:

$$\frac{\partial^2 \Phi}{\partial x^2} + B^2 \Phi = 0 \tag{7-7}$$

- The neutron flux within the reactor will be determined using boundary condition
- The neutron flux vanishes on extrapolated surface $\tilde{a} = a + 2d$
- The boundary condition becomes:

$$\Phi\left(\frac{\tilde{a}}{2}\right) = \Phi\left(-\frac{\tilde{a}}{2}\right) = 0$$

- It is also obvious that because of the problem symmetry ($\Phi(x) = \Phi(-x)$), there will be maximum neutron flux density and no net flow in the reactor centre:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}x} = 0 \text{ , for } x = 0$$

Slab Reactor (2)

- General solution to equation (7-7) is:

$$\Phi(x) = A\cos(Bx) + C\sin(Bx) \tag{7-8}$$

- Constant A and C are to be determined by boundary conditions
- Placing the derivative of (7-8) equal zero at x = 0 gives immediately C = 0
- The general solution reduces to:

$$\Phi(x) = A\cos(Bx) \tag{7-9}$$

- Introducing the boundary condition gives:

$$\Phi\left(\frac{\tilde{a}}{2}\right) = A\cos\left(\frac{B\tilde{a}}{2}\right) = 0$$

Slab Reactor (3)

- Solution to the above equation is either trivial A = 0, or must satisfy equation:

$$\cos\left(\frac{B\tilde{a}}{2}\right) = 0$$

- There are B_n solutions:

$$B_n = \frac{n\pi}{\tilde{a}}, \text{ n is odd integer}$$

- The various B_n constants are known as *eigenvalues*
- It can be shown that only the first eigenfunction is solution of neutron flux in a steady state critical reactor
- Function describing neutron flux in a bare critical slab reactor is:

$$\Phi(x) = A\cos B_1 x = A\cos\left(\frac{\pi x}{\tilde{a}}\right)$$

Reactor Buckling

- The square of the lowest eigenvalue B_1^2 is called *reactor buckling*
- It is found by solving the equation

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} + B_1^2\Phi = 0$$

$$B_1^2 = -\frac{1}{\Phi}\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2}$$

- The right-hand side is proportional to the curvature of the neutron flux in the reactor
- Since in the slab reactor:

$$B_1^2 = \left(\frac{\pi}{\widetilde{a}}\right)^2$$

- Buckling decreases as $\widetilde{a}$ is increasing
- In the limit, B_1^2 is approaching zero and flux is constant and does not buckle

Neutron Flux Magnitude

- The constant A has not been determined yet. It is related to the reactor power and not material composition
- Reactor power is determined as $P = VE_f\Sigma_f\Phi_{AV}$
- With recoverable energy from fission 200 MeV (3.2×10^{-11} J), the total power of the slab reactor can be calculated as:

$$P = E_f\Sigma_f \int_{-a/2}^{a/2} \Phi(x)\,\mathrm{d}x$$

- The integration is carried out for *physical* dimensions of the reactor
- Inserting the previously calculated function for Φ (7-9) and performing the integration gives:

$$P = \frac{2\tilde{a}E_f\Sigma_f A \sin\left(\frac{\pi a}{2\tilde{a}}\right)}{\pi}$$

Neutron Flux in a Bare Slab Reactor

- The final formula for neutron flux in a bare slab reactor is:

$$\Phi(x) = \frac{\pi P}{2\tilde{a} E_f \Sigma_f \sin\left(\frac{\pi a}{2\tilde{a}}\right)} \cos\left(\frac{\pi x}{\tilde{a}}\right) \tag{7-10}$$

- If d is small compared to physical dimensions of the reactor, the above formula reduces to:

$$\Phi(x) = \frac{\pi P}{2 a E_f \Sigma_f} \cos\left(\frac{\pi x}{a}\right) \tag{7-11}$$

Sphere Reactor

- The spherical reactor has radius R and neutron flux inside the reactor depends only on distance r
- Reactor equation (7-4) in spherical coordinates is:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\Phi}{\mathrm{d}r} + B^2\Phi = 0 \qquad (7\text{-}12)$$

- The neutron flux must satisfy boundary condition $\Phi(\tilde{R}) = 0$
- By substituting $\Phi = u/r$ into (7-12) and solving the resulting equation for u leads to a general solution for neutron flux:

$$\Phi(r) = A\frac{\sin(Br)}{r} + C\frac{\cos(Br)}{r}$$, where A and C are constants

- The second term becomes infinite when r goes to zero, thus C must be zero and resulting equation is:

$$\Phi(r) = A\frac{\sin(Br)}{r}$$

Sphere Reactor (2)

- Boundary condition is satisfied if B is one of the eigenvalues

$$\sin(B\widetilde{R}) = 0 \Longrightarrow B_n = \frac{n\pi}{\widetilde{R}}, \text{ where n is any integer}$$

- Only the first eigenvalue is relevant for a critical reactor, thus buckling is

$$B_1^2 = \left(\frac{\pi}{\widetilde{R}}\right)^2$$

- Flux becomes:

$$\Phi(r) = A\frac{\sin(\pi r/\widetilde{R})}{r} \tag{7-13}$$

Sphere Reactor (3)

- Constant A is determined by the reactor power

$$P = E_f \Sigma_f \int_V \Phi(r)\,\mathrm{d}V$$

- Volume element is calculated as $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$ and previous equation changes to:

$$P = 4\pi E_f \Sigma_f \int_0^R r^2 \Phi(r)\,\mathrm{d}r$$

- Introducing neutron flux (7-13) and integration leads to:

$$P = 4\pi E_f \Sigma_f A \frac{\widetilde{R}}{\pi} \left[\frac{\widetilde{R}}{\pi} \sin\left(\frac{\pi R}{\widetilde{R}}\right) - R \cos\left(\frac{\pi R}{\widetilde{R}}\right) \right]$$

- If d is small, neutron flux can be written in form:

$$\Phi(r) = \frac{P}{4 E_f \Sigma_f R^2} \frac{\sin(\pi r / R)}{r} \qquad \text{(7-14)}$$

Infinite Cylinder Reactor

- Consider a critical infinite bare cylinder with radius R
- In such a system, neutron flux depends only on the distance *r* from the cylinder axis
- The reactor equation (7-4) in cylindrical coordinates is:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\Phi}{\mathrm{d}r} + B^2\Phi = 0 \qquad \text{(7-15)}$$

- Neutron flux must satisfy not only this equation, but also the boundary condition $\Phi(\widetilde{R}) = 0$
- The equation (7-15) is an ordinary Bessel equation of the order zero. Its general solution is in form of the zero order ordinary Bessel functions of the first (J_0) and second (Y_0) kind:

$$\Phi(r) = AJ_0(Br) + CY_0(Br)$$

- Function Y_0 is infinite at r = 0, thus constant C must equal zero

Infinite Cylinder Reactor (2)

- Neutron flux reduces to:

$$\Phi(r) = AJ_0(Br)$$

- Based on function $J_0(x)$ shape, boundary condition $\Phi(\widetilde{R}) = AJ_0(B\widetilde{R}) = 0$ is satisfied at several values of x_n
- It means that the boundary condition is satisfied if B is one the values:

$$B_n = \frac{x_n}{\widetilde{R}}$$

- Again, only the first eigenvalue is valid for a critical reactor and buckling must equal:

$$B_1^2 = \left(\frac{x_1}{\widetilde{R}}\right)^2 = \left(\frac{2.405}{\widetilde{R}}\right)^2 \tag{7-16}$$

Infinite Cylinder Reactor (3)

- One-group flux in an infinite bare reactor is

$$\Phi(r) = AJ_0\left(\frac{2.405r}{\tilde{R}}\right) \tag{7-17}$$

- The constant A is determined from the reactor power
- Volume element in cylindrical reactor is $\mathrm{d}V = 2\pi r \mathrm{d}r$, thus power per unit length of the reactor is:

$$P = 2\pi E_f \Sigma_f \int_0^R r\Phi(r)\,\mathrm{d}r = 2\pi E_f \Sigma_f \int_0^R rJ_0\left(\frac{2.405r}{\tilde{R}}\right)\,\mathrm{d}r \tag{7-18}$$

Infinite Cylinder Reactor (4)

- The equation (7-18) can be evaluated using formula $\int J_0(x')x'\,\mathrm{d}x' = xJ_1(x)$, which gives for small d

$$P = 2\pi E_f \Sigma_f R^2 A \frac{J_1(2.405)}{2.405} = 1.35 E_f \Sigma_f R^2 A$$

- Final expression for the neutron flux is:

$$\Phi(r) = \frac{0.738P}{E_f \Sigma_f R^2} J_0\left(\frac{2.405r}{R}\right) \tag{7-19}$$

Finite Cylinder Reactor

- Finite cylinder reactor with radius R and height H is close to a real reactor system
- In this reactor, the neutron flux depends on the distance r from the axis and the distance z from the midpoint of the cylinder
- The reactor equation (7-4) is following:

$$\frac{\partial^2 \Phi(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi(r,z)}{\partial r} + \frac{\partial^2 \Phi(r,z)}{\partial z^2} + B^2 \Phi(r,z) = 0$$

- The solution must satisfy boundary conditions $\Phi(\widetilde{R}, z) = 0$ and $\Phi(r, \widetilde{H}/2) = 0$
- The solution is obtained by assuming separation of variables $\Phi(r,z) = R(r)Z(z)$

Finite Cylinder Reactor(2)

- Separation of variables leads to two equations, which can be solved independently:

$$\frac{\mathrm{d}^2 R}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}R}{\mathrm{d}r} + B_r^2 R = 0$$

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}z^2} + B_z^2 Z = 0$$

- The buckling $B^2 = B_r^2 + B_z^2$
- The final solution and application of boundary conditions is similar to the previously solved slab and infinite cylinder reactors:

$$\Phi(r,z) = AJ_0\left(\frac{2.405r}{\widetilde{R}}\right)\cos\left(\frac{\pi z}{\widetilde{H}}\right) \tag{7-20}$$

- $\widetilde{H} = H + 2d$ and $\widetilde{R} = R + d$ are extrapolated boundaries
- Constant A can be determined from the reactor power

Maximum to Average Flux and Power

- Maximum neutron flux (reactor power) is found in the centre of the reactor
- It is useful to known the ratio of the average neutron power to the maximum value
- For example, in a spherical nuclear reactor the maximum flux is:

$$\Phi_{max} = \frac{P}{4E_f\Sigma_f R^2} \lim_{r\to 0} \frac{\sin(\pi r/R)}{r} = \frac{\pi P}{4E_f\Sigma_f R^3}$$

- Average flux is calculated as:

$$\Phi_{av} = \frac{P}{E_f\Sigma_f V}$$

- Dividing the previous equations gives power peaking factor K for a sphere reactor

$$K = \frac{\Phi_{max}}{\Phi_{av}} = \frac{\pi^2}{3} = 3.29$$

- Power distribution is kept uniform in the reactor core

Critical Equations

- It was shown previously that it is necessary for a reactor to be critical that B^2 must be equal to the first eigenvalue B_1^2
- Critical equations will be determined for one-group and two-group reactor equations

One-Group Critical Equation

- Critical reactor follows equation (7-5), which solved for B^2 gives:

$$B^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$$, or the critical buckling must be:

$$B_c^2 = \frac{\frac{\nu\Sigma_f}{\Sigma_a} - 1}{\frac{D}{\Sigma_a}}$$

- The right-hand side must equal to the first eigenvalue B_1^2 depending only on dimensions and geometry of the reactor
- Using definition of k_∞ and L^2 it can be written:

$$\frac{k_\infty - 1}{L^2} = B_1^2$$

- It can be interpreted that in order to reach critical state, physical dimensions and geometry must by appropriate to fuel composition

Neutron Leakage

- The previous equation can be rearranged in the following form (from this moment B^2 refers to buckling of a critical reactor):

$$\frac{k_\infty}{1 + B^2 L^2} = 1 \tag{7-21}$$

- This form is usually known as *one-group critical equation*

- Rate of neutron leakage from volume V through area A is:

$$\int_A \boldsymbol{J}\boldsymbol{n}\, \mathrm{d}A = \int_V div \boldsymbol{J}\, \mathrm{d}V = -D \int_V \nabla^2 \Phi\, \mathrm{d}V$$

- From reactor equation (7-4), this can be rewritten as:

$$-D \int_V \nabla^2 \Phi\, \mathrm{d}V = DB^2 \int_V \Phi\, \mathrm{d}V$$

Neutron Leakage (2)

- Neutrons can either escape from the reactor or be absorbed inside with no other alternative
- Probability of *neutron non-leakage* (P_{NL}) can be calculated as:

$$P_{NL} = \frac{\Sigma_a \int_V \Phi \, \mathrm{d}V}{\Sigma_a \int_V \Phi \, \mathrm{d}V + DB^2 \int_V \Phi \, \mathrm{d}V} = \frac{\Sigma_a}{\Sigma_a + DB^2} = \frac{1}{1 + B^2 L^2} \tag{7-22}$$

- From comparing previous equations (7-21) and (7-22) it can be concluded that critical equation can be rewritten in form:

$$k_\infty P_{NL} = 1 \tag{7-23}$$

- This result has following interpretation

Neutron Leakage (3)

- Total $\Sigma_a \int_V \Phi \, dV$ neutrons are absorbed in the reactor every second and this leads to release of fission neutrons:

$$\eta f \Sigma_a \int_V \Phi \, dV = k_\infty \Sigma_a \int_V \Phi \, dV$$

- Due to leakage, only $P_{NL} k_\infty \Sigma_a \int_V \Phi \, dV$ initiate new generation of neutrons
- From definition of multiplication factor follows that:

$$k = \frac{P_{NL} k_\infty \int_V \Sigma_a \Phi \, dV}{\Sigma_a \int_V \Phi \, dV} = k_\infty P_{NL} = \eta f P_{NL} \tag{7-24}$$

- Thus, the left-hand side of the critical equation (7-21) is actually the multiplication factor for the reactor
- The critical equation follows by placing k = 1

Thermal Reactors

- Most of the currently operated nuclear reactors are thermal reactors
- Thermal reactors can consist of fuel, constructional materials, moderator, and coolant
- Only fuel and moderator will be included in the following analysis
- All materials apart from fuel are considered as moderator
- Purpose of moderator is to slow-down neutrons to reach thermal energy range

Thermal Reactors Difference

- Previous criticality calculations were performed for one neutron group – suitable for fast reactors
- One-group method is not sufficient for calculation of thermal reactors, because neutrons can diffuse for a considerable distance while slowing down
- At least two neutron groups must be considered:
  - fast group for neutrons released during fission
  - thermal group for neutrons at energies with high probabilities to initiate fission
- It can be assumed that there is no absorption in the fast group and in this group neutrons are only lost as a result of slowing-down into the thermal group

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Two-Group Critical Calculation

- $\Sigma_1\Phi_1$ neutrons are scattered out of the fast group per cm^3/sec, where Φ_1 is fast neutron flux
- Fission is initiated mostly by thermal neutrons, the few fissions by fast neutrons are taken into account by *fast fission factor*
- It follows that $\eta_T f \varepsilon \bar{\Sigma}_a \Phi_T = (k_\infty/p)\bar{\Sigma}_a\Phi_T$ fission neutrons are emitted per cm^3/sec
- These neutrons appear in the fast group as a source of neutrons
- Formulation of reactor equations in two energy groups are following:

$$D_1\nabla^2\Phi_1 - \Sigma_1\Phi_1 + \frac{k_\infty}{p}\bar{\Sigma}_a\Phi_T = 0, \text{ fast group} \qquad \text{(7-25)}$$

$$\bar{D}\nabla^2\Phi_T - \bar{\Sigma}_a\Phi_T + p\Sigma_1\Phi_1 = 0, \text{ thermal group} \qquad \text{(7-26)}$$

Two-Group Critical Calculation (2)

- Both thermal and fast neutron flux follow the same spatial distribution and can be written as:

$$\Phi_1 = A_1 \Phi$$

$$\Phi_T = A_2 \Phi$$

- A_1 and A_2 are constants and Φ satisfies the equation:

$$\nabla^2 \Phi + B^2 \Phi = 0$$

- Substituting the last three equation into (7-25) and (7-26) yields:

$$-(D_1 B^2 + \Sigma_1) A_1 + \frac{k_\infty}{p} \bar{\Sigma}_a A_2 = 0 \qquad (7\text{-}27)$$

$$p \Sigma_1 A_1 - (\bar{D} B^2 + \bar{\Sigma}_a) A_2 = 0 \qquad (7\text{-}28)$$

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Two-Group Critical Calculation (3)

- According to Cramer's rule equations (7-27) and (7-28) have non-trivial solution if system determinant equals zero:

$$\begin{vmatrix} -(D_1 B^2 + \Sigma_1) & \frac{k_\infty}{p}\bar{\Sigma}_a \\ p\Sigma_1 & -(\bar{D}B^2 + \bar{\Sigma}_a) \end{vmatrix} = 0$$

- Multiplying out the determinant gives:

$$k_\infty \bar{\Sigma}_a \Sigma_1 - (\bar{D}B^2 + \bar{\Sigma}_a)(D_1 B^2 + \Sigma_1) = 0$$

- Rearranging and dividing by $\Sigma_1 \bar{\Sigma}_a$ finally yields:

Two-group critical equation for a bare homogeneous reactor

$$\frac{k_\infty}{(1 + B^2 L_T^2)(1 + B^2 \tau_T)} = 1 \qquad \text{(7-29)}$$

Two-Group Critical Equation

- In the previous equation (7-29) were used L_T^2 – *thermal diffusion area* and *neutron age* – τ_T defined as:

$$L_T^2 = \frac{\bar{D}}{\bar{\Sigma}_a} \text{ and } \tau_T = \frac{D_1}{\Sigma_1}$$

- The two-group critical equation (7-29) contains probability that thermal neutron will not leak from the reactor – P_{TNL} and probability of not escaping while slowing-down – P_{FNL}

$$P_{TNL} = \frac{1}{1 + B^2 L_T^2} \text{ and } P_{FNL} = \frac{1}{1 + B^2 \tau_T}$$

- Multiplication factor of thermal reactor is $k = k_\infty P_{TNL} P_{FNL}$
- If denominator in the two-group critical equation is multiplied out, term $B^4 L_T^2 \tau_T$ can often be ignored resulting in expression:

$$\frac{k_\infty}{1 + B^2(L_T^2 + \tau_T)} = 1 \qquad (7\text{-}30)$$

Modified One-Group Critical Equation

- It is possible to define *thermal migration area* – $M_T^2 = L_T^2 + \tau_T$, then equation (7-30) can be written as:

$$\frac{k_\infty}{1 + B^2 M_T^2} = 1 \qquad \text{(7-31)}$$

- This form is known as *modified one-group critical equation*

- It is clear that if τ_T is much smaller than L_T^2, then the reactor is sufficiently described by the one-group critical equation
- This approach is suitable for graphite and D_2O moderated reactor
- Two-group theory is necessary for reactors moderated by H_2O
- The resulting equations (7-29) and (7-31) are used to calculate critical mass of bare thermal reactors

Reactor Reflector

- Neutron economy is improved if the reactor core is surrounded by a reflector
- Reflector is realized by a thick unfueled region of moderator
- Neutrons escaping from the core must pass through this moderator region and some of these diffuse back
- The net result is that critical dimensions of the reactor are reduced
- Reflector parameters will be derived using one-group method for a sphere reactor of radius R
- The following analysis deals only with an infinite reflector

Reflected Sphere Reactor

- In the following analysis, parameters referring to the reactor core and reflector will have subscripts *c* and *r*, respectively
- Neutron flux in the reactor core must follow the equation:

$$\nabla^2\Phi_c + B^2\Phi_c = 0 \text{ , where for critical reactor} \tag{7-32}$$

$$B^2 = \frac{k_\infty - 1}{L_c^2}$$

- Since there are no neutron sources in the reflector, neutron flux in this area must satisfy the following equation:

$$\nabla^2\Phi_r - \frac{1}{L_r^2}\Phi_r = 0 \tag{7-33}$$

- The previous equations for Φ_c and Φ_r must be solved with respect to boundary conditions

Reflected Sphere Reactor (2)

- General solution for equation (7-32) was derived previously and it is in form:

$$\Phi_c = A_c \frac{\sin(Br)}{r} + C_c \frac{\cos(Br)}{r}$$

- Because neutron flux must be finite in the whole reactor volume (including r = 0), constant C_c must equal 0 and Φ_c reduces to:

$$\Phi_c = A_c \frac{\sin(Br)}{r} \qquad (7\text{-}34)$$

- General solution to equation (7-33) is:

$$\Phi_r = A_r \frac{e^{-r/L_r}}{r} + C_r \frac{e^{r/L_r}}{r}$$

- Constant C_r must equal zero to keep neutron flux finite as *r* goes to infinity and the equation reduces to:

$$\Phi_r = A_r \frac{e^{-r/L_r}}{r} \qquad (7\text{-}35)$$

Reflected Sphere Reactor (3)

- Boundary conditions for neutron flux and neutron current on the core/reflector interface (r = R) are used:

$$\Phi_c(R) = \Phi_r(R) \text{ and} \tag{7-36}$$

$$-D_c\left(\frac{\mathrm{d}\Phi_c}{\mathrm{d}r}\right)_R = -D_r\left(\frac{\mathrm{d}\Phi_r}{\mathrm{d}r}\right)_R \tag{7-37}$$

- Introducing equations (7-34) and (7-35) into (7-36) gives:

$$A_c\frac{\sin(BR)}{R} = A_r\frac{e^{-R/L_r}}{R} \tag{7-38}$$

- Next, differentiating equations (7-34) and (7-35) and inserting results into (7-37) yields:

$$A_cD_c\left(\frac{B\cos(BR)}{R} - \frac{\sin(BR)}{R^2}\right) = -A_rD_r\left(\frac{1}{RL_r} + \frac{1}{R^2}\right)e^{-R/L_r} \tag{7-39}$$

Critical Equation of a Reflected Sphere Reactor

- System of equations (7-38) and (7-39) for unknowns A_c and A_r has nontrivial solution if determinant of the system equals zero. This is satisfied if

$$D_c \left(B \cot(BR) - \frac{1}{R} \right) = -D_r \left(\frac{1}{L_r} + \frac{1}{R} \right) \tag{7-40}$$

- The equation (7-40) is usually rearranged in the following form:

$$BR \cot(BR) - 1 = -\frac{D_r}{D_c} \left(\frac{R}{L_r} + 1 \right) \tag{7-41}$$

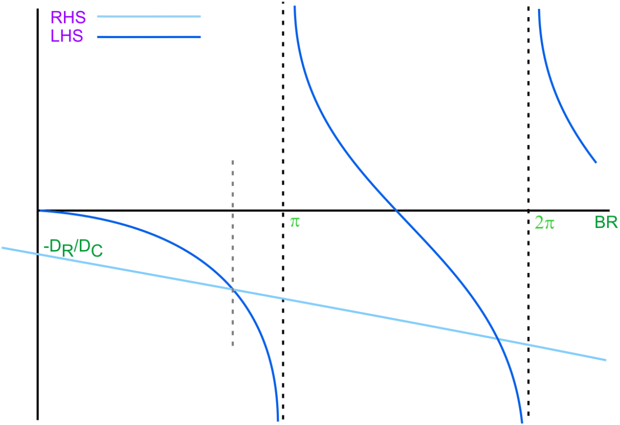
- It is *critical equation for sphere reactor with infinite reflector*

- It must be satisfied for a reactor to be critical
- For given R, B^2 must be calculated from equation (7-41) and critical composition can be determined

Solution of Critical Equation of a Reflected Sphere Reactor

- Critical equation (7-41) is transcendental and cannot be solved analytically, but graphical solution is possible
- Suppose that the core composition and hence B is specified
- Reactor core radius can then be found by plotting the left-hand side (LHS) and the right-hand side (RHS) of the equation separately as functions of BR
- As shown later, LHS has infinite number of branches and RHS forms a straight line with slope $-D_r/(D_c B L_r)$
- Every value of BR corresponding to intersection of LHS and RHS forms an infinite number of solutions
- Only the first solution is relevant to a critical reactor
- It is to be noted that the solution is smaller that π, which means that critical radius of reflected sphere is smaller than for a bare spherical reactor

Graphical Solution of Critical Equation of a Reflected Sphere Reactor



Graphical solution to the critical equation of a reflected sphere reactor

Special Solution of Critical Equation of a Reflected Sphere Reactor

- In a special case in which moderator in the core and reflector are the same, $D_c = D_r$ and equation (7-41) reduces to:

$$B \cot(BR) = -\frac{1}{L_r} \tag{7-42}$$

- This equation is not transcendental in R, thus if B is known, critical radius R can be calculated directly

Determination of Power of a Critical Reflected Sphere Reactor

- Having obtained the criticality conditions, it is necessary to evaluate constants A_c and A_r from the reactor power
- Constants A_c and A_r are related by equation (7-38) and it is possible to find for instance A_r in terms of A_c

$$A_r = A_c e^{R/L_r} \sin(BR)$$

- The reactor power is determined as $P = E_f \Sigma_f \int_V \Phi_c \,\mathrm{d}V$ with $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$

$$P = 4\pi E_f \Sigma_f A_c \int_0^R r \sin(BR) \mathrm{d}r = \frac{4\pi E_f \Sigma_f A_c}{B^2} (\sin(BR) - BR\cos(BR))$$

- Solution for A_c gives:

$$A_c = \frac{PB^2}{4\pi E_f \Sigma_f (\sin(BR) - BR\cos(BR))}$$

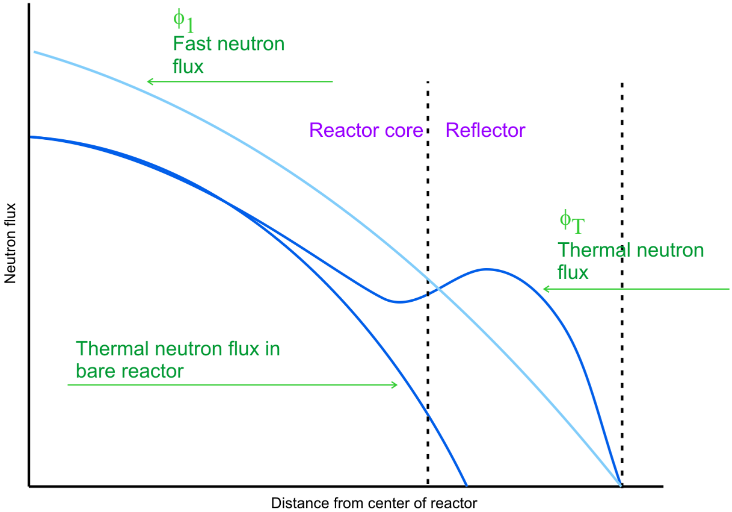
Reflected Reactor Discussion

- The presented procedure of critical equation determination is analytically possible only for spherical and fully reflected parallelepiped and cylindrical reactor
- In practise, critical conditions of these reactors are evaluated by transforming to a spherical reactor of the same composition and volume
- The whole calculation was carried using one-group method without regard for whether the reactor is fast or thermal
- It is therefore valid for both energy groups
- One-group method is sufficient for critical parameters computation, but for neutron flux distribution it is necessary to use two-group calculation

Flux in a Reflected Thermal Reactor

- In two-group approximation, it is necessary to solve a two-group reactor equation for the reactor core and a two-group diffusion equation for the reflector
- The evaluation will not be presented here, only the most important results
- The most striking result is that thermal neutron flux rises near the core-reflector interface and exhibits a peak in the reflector
- It is caused by fast neutrons thermalisation in the reflector
- Thermal neutrons are not much absorbed in the reflector, therefore they tend to accumulate there before they escape through the outer surface or diffuse back into the core
- This leads to flattening of the neutron flux distribution in the core

Plotted Neutron Flux a in Reflected Thermal Reactor



Reflector Savings

- Reactor reflector reduces critical dimensions of the core and reduces maximum-to-average power ratio
- Reflector savings is defined as:

$$\delta = \widetilde{R}_0 - R \text{ , where } \widetilde{R}_0 \text{ is critical diameter of a bare reactor} \quad (7\text{-}43)$$

- Following formula can be used to roughly estimate reflector savings for D_2O and graphite moderated/reflected reactor:

$$\delta \simeq \frac{\bar{D}_c}{\bar{D}_r} L_{Tr} \quad (7\text{-}44)$$

- The equation (7-44) is valid if reflector is several diffusion lengths thick, then it can be considered effectively infinite and further thickness increase does not reduce critical size of the core
- In practice all reactors have sufficiently thick reflectors to be considered infinite

Reason for Heterogeneous Reactors

- Usual configuration of a nuclear reactor is not homogeneous
- There are two typical configurations:
  - quasihomogeneous
    - mean free path for all neutrons is greater than fuel dimensions
    - it can be considered as homogeneous for neutrons
  - heterogeneous

Heterogeneous Reactors

- Mean free path for some neutron energies is smaller than fuel dimensions
- Several interactions are possible for a single neutron in the fuel region
- Regions of fuel and moderator must be solved individually
- Analytical solution is usually not possible for heterogeneous reactors
- It is possible to compare four factors in infinite multiplication factor
- The major difference can be found for thermal utilization factor and resonance escape probability

Thermal Utilization Factor

- It gives probability of thermal neutron absorption in fuel region
- Lets calculate number of neutrons absorbed in fuel region per second:

$$\int_{V_F} \overline{\Sigma}_a^F \Phi_T \, \mathrm{d}V = \overline{\Sigma}_a^F \overline{\Phi}_T^F V_F$$

- $\overline{\Phi}_T^F$ is average thermal neutron flux in fuel region, V_F is fuel region volume
- The same applies to absorption rate of thermal neutrons in moderator region, and possibly other materials:

$$\overline{\Sigma}_a^M \overline{\Phi}_T^M V_M$$

Disadvantage Factor

- It is possible to calculate from thermal utilization factor definition:

$$f = \frac{\overline{\Sigma}_a^F \overline{\Phi}_T^F V_F}{\overline{\Sigma}_a^F \overline{\Phi}_T^F V_F + \overline{\Sigma}_a^M \overline{\Phi}_T^M V_M} \tag{7-45}$$

- Lets introduce this ratio $\overline{\Phi}_T^M / \overline{\Phi}_T^F$ into the previous equation:

$$f = \frac{\overline{\Sigma}_a^F V_F}{\overline{\Sigma}_a^F V_F + \overline{\Sigma}_a^M V_M \dfrac{\overline{\Phi}_T^M}{\overline{\Phi}_T^F}} \tag{7-46}$$

- Ratio $\overline{\Phi}_T^M / \overline{\Phi}_T^F > 1$ is called disadvantage factor
- There is higher neutron absorption in fuel region

Thermal Utilization Factor

- Value of infinite multiplication factor depends on product of resonance escape probability and thermal utilization factor definition
- There is the same neutron flux in fuel and moderator regions in homogeneous reactors
- It means that using the equation(7-46) we can find that:

$$f_{HET} < f_{HOM} \tag{7-47}$$

Self-Shielding Factor

- There are neutron flux depressions due to increased neutron absorption in some energy groups in fuel
- It is difficult for neutrons in these energy groups to penetrate deep in fuel regions
- This effect leads to self-shielding of inner fuel parts
- It is the most important for thermal and resonance neutrons
- These neutron types are getting in the fuel region from moderator region
- On the average, fuel in heterogeneous reactors is exposed to lower neutron fluxes that in homogeneous reactors

Resonance Escape Probability

- The self-shielding effect reduces thermal utilization factor but it influences more the resonance escape probability
- There is increased probability to escape resonance absorption
- Possibility to slow-down below the resonance region is increased in heterogeneous reactors

$$p_{\mathrm{HET}} > p_{\mathrm{HOM}} \tag{7-48}$$

Final Effect of Heterogeneous Design

- Decrease of thermal utilization factor is more than compensated by increase of resonance escape probability
- It can be concluded that:

$$(fp)_{HET} > (fp)_{HOM} \tag{7-49}$$

- Practical consequence is that it is possible to design a critical heterogeneous reactor composed from natural uranium and graphite
- Homogeneous k_∞ of such a mixture is only 0.85