Diffusion Theory

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Introduction

- It is necessary to predict neutron distribution inside a nuclear reactor
- The exact description of all the processes of neutrons (collisions, transport, nuclear reactions) is very difficult
- The first approximation describes the movement of neutrons as a kind of diffusion
- This approximation is called *diffusion approximation* and was used in development of the first types of nuclear reactors
- More advanced methods are developed now, but still, diffusion theory is widely used for the analysis of large nuclear reactors
- The complete theory describing all neutron properties with little approximation is *Transport theory* solving *Boltzmann transport equation*

Fick’s Law

- The diffusion theory is based on *Fick’s law* originally used for chemical diffusion
- It was observed in chemistry that if concentration of a solute in one region of solution is greater than in another, the solute diffuses from the region of higher concentration to the region of lower concentration
- The rate of solute flow is proportional to the negative of the gradient of the solute concentration
- Neutrons behave to a good approximation in the same way
- If the density (neutron flux) of neutrons is higher in one part of a reactor, there is a net flow of neutrons into a region with lower neutron flux

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Neutron Current Density

- If neutron density varies along x-direction, the net flow of neutrons that pass per unit of time through a unit area perpendicular to the x-direction can be expressed as:

$$J_x = -D\frac{\mathrm{d}\Phi}{\mathrm{d}x} \tag{6-1}$$

- J_x has the same unit as flux (neutrons/cm^2-sec)
- Parameter D is called *diffusion coefficient* and has unit of cm
- The flux is generally function of three spatial variables, therefore:

$$\boldsymbol{J} = -D\,\mathrm{grad}\Phi = -D\,\nabla\Phi \tag{6-2}$$

- Here **J** is called *neutron current density*

Diffusion Coefficient

- We assume that D is not a function of spatial variables
- The diffusion coefficient can be approximately calculated as:

$$D = \frac{\lambda_{\text{tr}}}{3} \text{, where } \lambda_{\text{tr}} \text{ is transport mean free path} \tag{6-3}$$

$$\lambda_{\text{tr}} = \frac{1}{\Sigma_{\text{tr}}} = \frac{1}{\Sigma_s(1-\bar{\mu})} \tag{6-4}$$

- Transport mean free path (λ_{tr}) is an average distance a neutron will move in its original direction after infinite number of collisions
- $\bar{\mu} = \overline{\cos}\vartheta$ is average value of the cosine of the angle at which neutrons are scattered in the medium. It can be calculated for most of the neutron energies as:

$$\bar{\mu} = \frac{2}{3A} \tag{6-5}$$

Validity of Fick's Law

Fick's law is approximation which is not valid under the following conditions

1. In a medium that strongly absorbs neutrons
2. Within about three mean free paths from either neutron source or the outer surface of the diffusive medium
3. When the scattering of neutrons is strongly anisotropic

- These conditions are present in every practical reactor problem
- Fick's law and diffusion theory is therefore only the first estimate
- More advanced methods must be used near sources, boundaries and in strongly absorbing media

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General Equation of Continuity

- In an arbitrary volume V of a diffusive medium containing neutrons the number of neutrons may change
- The change of the number of neutrons is a result of a net flow of neutrons in or out of V, some neutrons are absorbed inside V and there might be also neutron sources inside volume V
- The *equation of continuity* is mathematical representation of the fact that neutrons cannot disappear unaccountably

$$\begin{bmatrix}\text{rate of change}\\ \text{in number of}\\ \text{neutrons in V}\end{bmatrix} = \begin{bmatrix}\text{rate of}\\ \text{production}\\ \text{of neutrons}\\ \text{in V}\end{bmatrix} - \begin{bmatrix}\text{rate of}\\ \text{absorption}\\ \text{of neutrons}\\ \text{in V}\end{bmatrix} - \begin{bmatrix}\text{rate of}\\ \text{leakage}\\ \text{of neutrons}\\ \text{from V}\end{bmatrix}$$

Rate of Change of Neutrons in V

- If n is density of neutrons at any point in time in V, the total number of neutrons in V is then:

$$\int_V n \, \mathrm{d}V$$

- The rate of change in number of neutrons is:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V n \, \mathrm{d}V \text{ , which can be also written as:}$$

$$\int_V \frac{\partial n}{\partial t} \, \mathrm{d}V$$

Production and Absorption Rate in V

- Let s be the rate at which neutrons are emitted from sources per cm^3/sec in V
- The rate at which neutrons are produced through V is given as:

$$\text{Production rate} = \int_V s \,\mathrm{d}V$$

- The rate at which neutrons are lost by absorption per cm^3/sec is equal to $\Sigma_a \Phi$
- The total loss of neutrons through absorption in volume V is:

$$\text{Absorption rate} = \int_V \Sigma_a \Phi \,\mathrm{d}V$$

Leakage Rate out of V

- **J** is current density vector on the surface of V and **n** is a unit normal pointing outward from the surface
- Then **Jn** is the net number of neutrons passing outwards through the surface per cm^2/sec
- The overall leakage through the surface A of the volume V is:

$$\text{Leakage rate} = \int_A \mathbf{Jn}\,\mathrm{d}A$$

- This surface integral can be converted into a volume integral by the divergence theorem:

$$\int_A \mathbf{Jn}\,\mathrm{d}A = \int_V \mathrm{div}\mathbf{J}\,\mathrm{d}V$$

Resulting Equation of Continuity

$$\int_V \frac{\partial n}{\partial t}\,\mathrm{d}V = \int_V s\,\mathrm{d}V - \int_V \Sigma_a \Phi\,\mathrm{d}V - \int_V \mathrm{div}\mathbf{J}\,\mathrm{d}V$$

- Integral were carried out over the same volume, thus their integrands must also be equal:

$$\frac{\partial n}{\partial t} = s - \Sigma_a \Phi - \mathrm{div}\mathbf{J} \qquad (6\text{-}6)$$

- The above equation is the *equation of continuity*, if the neutron density is not a function of time, this equation reduces to:

$$-\mathrm{div}\mathbf{J} - \Sigma_a \Phi + s = 0$$

Diffusion Equation

- The continuity equation has two unknowns – the neutron density (n) and neutron current density (**J**)
- There is a relation between neutron flux and neutron current density
- One of these unknowns can be eliminated by Fick's law
- Substitution of (6-2) into (6-6) leads to:

$$-\mathrm{div}(-D\,\mathrm{grad}\Phi) - \Sigma_a\Phi + s = \frac{\partial n}{\partial t}$$

- Diffusion coefficient (D) is spatially independent

$$D\,\mathrm{div}(\mathrm{grad}\Phi) - \Sigma_a\Phi + s = \frac{\partial n}{\partial t}$$

Diffusion Equation (2)

- The continuity equation can be further simplified by introducing symbol $\nabla^2 \equiv \text{div grad}$ called *Laplacian operator*
- The resulting equation is called *diffusion equation*

$$D\nabla^2\Phi - \Sigma_a\Phi + s = \frac{1}{v}\frac{\partial\Phi}{\partial t} \tag{6-7}$$

- If only time independent problems are considered, *steady-state diffusion equation* is formulated

$$D\nabla^2\Phi - \Sigma_a\Phi + s = 0 \tag{6-8}$$

Laplacian Operator

- Formula for Laplacian depends on used coordinate system
  - in rectangular coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

  - in cylindrical coordinates:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2} + \frac{\partial^2}{\partial z^2}$$

  - in spherical coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta\frac{\partial}{\partial \vartheta}\right) + \frac{1}{r^2 \sin^2\vartheta}\frac{\partial^2}{\partial \varphi^2}$$

1D Laplacian Operators

- In the simplest examples in one-dimensional space, the Laplacian operator reduces to the following formulas:
  - rectangular coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial x^2}$$

  - cylindrical coordinates:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

  - spherical coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

Diffusion Length

- The equation (6-8) is often divided by D, resulting in:

$$\nabla^2\Phi - \frac{1}{L^2}\Phi + \frac{s}{D} = 0 \tag{6-9}$$

- Parameter L^2 is defined as:

$$L^2 = \frac{D}{\Sigma_a}$$

- The quantity L is called *diffusion length* with unit cm
- Quantity L^2 is diffusion area, unit cm^2
- Physical meaning of the diffusion length will be given later

Boundary Conditions

- The diffusion equation was derived using Fick's law, therefore conditions for validity of Fick's law are also valid for the diffusion equation
- Since the diffusion equation is a partial differential equation, boundary conditions are required
- There are typical boundary conditions:
  1. Neutron flux must be non-negative and finite:
     $0 \leqslant \Phi < \infty$
  2. Both neutron flux and current must be continuous across boundary of two diffusive media (A and B):

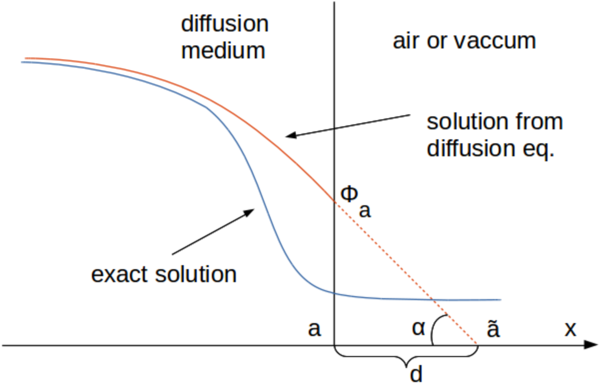
$$\Phi_A = \Phi_B$$
$$(J_A)_n = (J_B)_n$$

  3. Boundary condition for an external boundary of a diffusive medium
  4. Source conditions

Extrapolation Distance

- Fick's law is not valid for area close to an external surface between the diffusive medium and atmosphere
- It was found that if the flux vanishes in a distance d from the surface, then the flux calculated by diffusion theory is close to the real flux
- The parameter d is known as *extrapolation distance* and in most cases it is given by simple formula $d = 0.71\ \lambda_{tr}$, where λ_{tr} is transport mean free path of the medium
- From relation for diffusion coefficient – $D = \lambda_{tr}/3$ – results that $d = 2.13\ D$
- The extrapolation distance is usually in units of several cm and therefore it can be in many cases neglected and assumed that neutron flux diminishes at the physical boundary of the medium

Extrapolation Distance – Visualisation



If d is not negligible, physical dimensions of the reactor are increased by d and *extrapolated boundary* is formulated with dimension $\tilde{a} = a + d$

Source Condition

- The diffusion equation is not valid for the neutron source location, but it is necessary to match the magnitude of the neutron flux to the source intensity
- The source is surrounded by area for which it holds that all neutrons flowing through this area must come from the neutron source characterized by source emissivity S
- Formulation depends on the source geometry: planar (6-10a), point (6-10b), or line (6-10c)

$$\lim_{x \to 0} J(x) = \frac{S}{2} \tag{6-10a}$$

$$\lim_{r \to 0} 4\pi r^2 J(r) = S \tag{6-10b}$$

$$\lim_{r \to 0} 2\pi r J(r) = S \tag{6-10c}$$

- This condition will be illustrated by examples of neutron sources in diffusive media

Infinite Diffusive Medium

- Spatial distribution of neutron flux in an infinite media will be calculated using diffusion equation and boundary conditions
- Basic source geometries will be calculated – plane, point and line
- Only monoenergetic sources of neutrons are analysed

Planar Source in Infinite Diffusive Medium

- Planar source emitting S neutrons per cm^2/sec
- The flux is only function of distance from the plane, e.g. in x direction. The source location is not part of the analysed area
- The plane is located in $x = 0$ and there are two solution for positive ($x > 0$) and negative ($x < 0$) directions
- The diffusion equation (6-9) has the following form:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} - \frac{1}{L^2}\Phi = 0 \text{ , } x \neq 0$$

- Solution for $x > 0$ is expected in form:

$$\Phi(x) = e^{-\lambda x} \text{ , second derivative: } \frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} = \lambda^2 e^{-\lambda x}$$

- Substituted in the above equation leads to:

$$\lambda^2 e^{-\lambda x} = \frac{1}{L^2} e^{-\lambda x}$$

Planar Source in Infinite Diffusive Medium (2)

- There are two possible solution for $\lambda = \pm\frac{1}{L}$
- The diffusion equation for planar source has thus general solution with two constants to be determined by boundary conditions

$$\Phi(x) = Ae^{-x/L} + Ce^{x/L}$$

- The second term can be eliminated from the condition of finite neutron flux, then the equation reduces to:

$$\Phi(x) = Ae^{-x/L}$$

- The constant A is determined from the source condition
- From Fick's law:

$$J = -D\frac{\mathrm{d}\Phi}{\mathrm{d}x} = \frac{DA}{L}e^{-x/L}$$

Planar Source in Infinite Diffusive Medium (3)

- Source condition for the planar source:

$$\lim_{x\to 0} J(x) = \lim_{x\to 0} \frac{DA}{L} e^{-x/L} = \frac{DA}{L} = \frac{S}{2}$$

- This gives constant A

$$A = \frac{SL}{2D}$$

- The final formula for spatial dependence of neutron flux from the planar source in infinite medium is:

$$\Phi(x) = \frac{SL}{2D} e^{-x/L} \tag{6-11}$$

- The solution is valid for $x > 0$, but because of symmetry of the problem similar formulation could be obtained for negative x-direction

Point Source in Infinite Diffusive Medium

- Point source emitting S neutrons/sec isotropically in an infinite medium
- The source is located in the centre of a spherical coordinate system and neutron flux depends only on distance r from the source
- The diffusion equation (6-9) in spherical coordinate system becomes for $r \neq 0$:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\Phi}{\mathrm{d}r} - \frac{1}{L^2}\Phi = 0$$

- The equation is solved by introducing substitution $u(r) = r\Phi(r)$
- Substitution into the above equation results in:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} - \frac{1}{L^2}u = 0$$

Point Source in Infinite Diffusive Medium (2)

- The solution for function *u* is found in an identical way as in the case of the planar source:

$$u(r) = Ae^{-r/L} + Ce^{r/L}$$

- Transform to the original function gives:

$$\Phi(r) = A\frac{e^{-r/L}}{r} + C\frac{e^{r/L}}{r}$$

- Constants A and C must be determined from boundary conditions
- It is clear that if the neutron density flux must remain finite, C must equal zero
- The constant A is found from the source condition
- From Fick's law:

$$J = -D\frac{\mathrm{d}\Phi}{\mathrm{d}r} = DA\left(\frac{1}{rL} + \frac{1}{r^2}\right)e^{-r/L}$$

Point Source in Infinite Diffusive Medium (3)

- The source condition for point source is:

$$\lim_{r\to 0} 4\pi r^2 J(r) = \lim_{r\to 0} 4\pi DA\left(\frac{r}{L}+1\right)e^{-r/L} = S$$

- This gives constant A:

$$A = \frac{S}{4\pi D}$$

- Resulting equation for neutron flux distribution is following:

$$\Phi(r) = \frac{Se^{-r/L}}{4\pi Dr} \tag{6-12}$$

Line Source in Infinite Diffusive Medium

- Line source emitting S neutrons/sec per unit length in an infinite medium
- The source is located in the centre of a cylindical coordinate system and neutron flux depends only on distance *r* from the source
- The diffusion equation (6-9) in the cylindical coordinate system becomes for $r \neq 0$:

$$\frac{\mathrm{d}^2\Phi(r)}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r} - \frac{1}{L^2}\Phi(r) = 0$$

- The equation can be transformed by substitution u=r/L into modified Bessel's equation of the order zero:

$$u^2\frac{\mathrm{d}^2\Phi(u)}{\mathrm{d}u^2} + u\frac{\mathrm{d}\Phi(u)}{\mathrm{d}u} - u^2\Phi(u) = 0$$

Ordinary Bessel's Functions

- Bessel's equation is:

$$x^2 \frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} + x\frac{\mathrm{d}\Phi}{\mathrm{d}x} + (\alpha^2 x^2 - n^2)\Phi = 0 \text{ , where } \alpha \text{ and } n \text{ are constants}$$

- n is order of the equation, in practical problems it is usually zero
- General solution to Bessel's equation is:

$$\Phi(x) = AJ_n(\alpha x) + CY_n(\alpha x)$$

- Functions J_n and Y_n are called *ordinary Bessel's functions of the first and second kind*, respectively

Modified Bessel's Functions

- If α^2 is negative, Bessel's equation becomes:

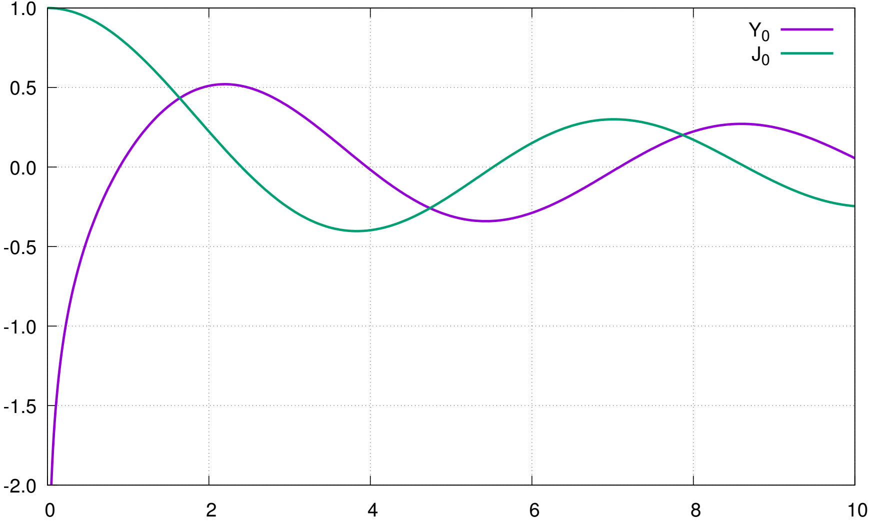
$$x^2\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} + x\frac{\mathrm{d}\Phi}{\mathrm{d}x} - (\alpha^2 x^2 + n^2)\Phi = 0$$

- General solution is in form of *modified Bessel's functions of the first and second kind*, respectively – I_n and K_n

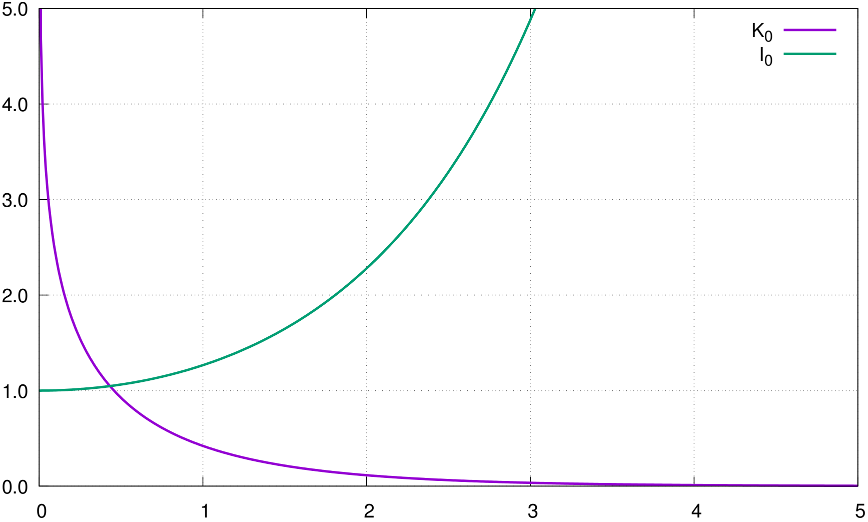
$$\Phi(x) = AI_n(\alpha x) + CK_n(\alpha x)$$

- The following figures will show Bessel's functions used in reactor physics

Ordinary Bessel's Functions Plot



Modified Bessel's Functions Plot



Line Source in Infinite Diffusive Medium (2)

- General solution of this kind of Bessel's equation is in form of modified Bessel's function of the first (I) and second (K) kind of the order zero

$$\Phi(u) = AI_0(u) + CK_0(u)$$

$$\Phi(r) = AI_0(r/L) + CK_0(r/L)$$

- Given the fact that function $I_0 \rightarrow \infty$ for $r \rightarrow \infty$, constant A must equal 0 and above equation reduces to:

$$\Phi(r) = CK_0(r/L)$$

- The constant C is found from the source condition
- From Fick's law, using $\mathrm{d}K_0/\mathrm{d}r = -K_1$:

$$J = -D\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -DC\frac{\mathrm{d}K_0(r/L)}{\mathrm{d}r} = \frac{DCK_1(r/L)}{L}$$

Line Source in Infinite Diffusive Medium (3)

- $K_1(x)$ behaves similar to $\frac{1}{x}$ function
- It can be used in the source condition:

$$\lim_{r\to 0} 2\pi (r/L) DCK_1(r/L) = S$$

- It can be written that $\lim_{r\to 0}[(r/L)K_1(r/L)] \sim 1$
- Resulting value for constant C is:

$$C = \frac{S}{2\pi D}$$

- Final form of a neutron flux distribution from the line source is:

$$\Phi(r) = \frac{S}{2\pi D} K_0(r/L) \tag{6-13}$$

The Diffusion Length

- It is necessary to understand physical meaning of the diffusion length L, which appears in solutions of neutron flux from neutron sources
- In diffusive medium, neutrons are moving along complicated paths, however, every neutron is absorbed in the medium since it is infinite
- Number of neutrons absorbed from a point neutron source per unit path length can be expressed as:

$$\mathrm{d}n = \Sigma_a \Phi(r)\,\mathrm{d}V = \frac{S\Sigma_a}{D} r e^{-r/L}\,\mathrm{d}r = \frac{S}{L^2} r e^{-r/L}\,\mathrm{d}r$$

- Probability that neutron is absorbed in dr is:

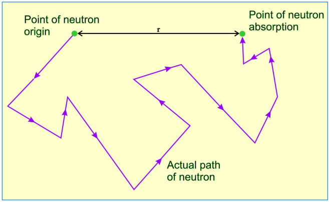
$$p(r)\,\mathrm{d}r = \frac{1}{L^2} r e^{-r/L}\,\mathrm{d}r$$

The Diffusion Length (2)

- It is now possible to calculate a square of an average distance from the source at which a neutron is absorbed

$$\bar{r^2} = \int_0^\infty r^2 p(r)\,\mathrm{d}r = \frac{1}{L^2}\int_0^\infty r^3 e^{-r/L}\,\mathrm{d}r = 6L^2$$

- It means that L^2 is one-sixth of the average of the square of the straight distance a neutron travels from the point at which it is emitted to the point where it is finally absorbed



- With higher diffusion length, neutrons are moving further. It means the diffusive medium is less absorbing

Finite Diffusive Medium

- Finite diffusive medium is a more realistic situation
- For neutron flux spatial distribution computation it is necessary to use boundary conditions
- For this purpose, physical dimensions of the diffusive media will be increased by extrapolation distance d
- Neutron flux will be calculated for monoenergetic neutron sources in basic geometries – slab, cylinder and sphere
- Since the method of solving these geometries is similar, it will be illustrated only for a bare slab with a plane source of neutrons

Finite Bare Slab

- The slab has thickness *2a*, it is infinite in vertical direction and the source of neutrons is located in its centre
- It can be solved for positive and negative direction with symmetrical results
- Boundary condition must be utilised
- The neutron flux is required to vanish at the extrapolated surface of the slab, at $x = a + d$ (or at $x = -d - a$)
- The boundary condition is:

$$\Phi(a + d) = \Phi(-a - d) = 0$$

- The general solution of the diffusion equation for the planar source of neutrons is the same as for infinite diffusive medium:

$$\Phi(x) = Ae^{-x/L} + Ce^{x/L}$$

Finite Bare Slab (2)

- Boundary condition at $a + d$ is applied:

$$\Phi(a+d) = Ae^{-(a+d)/L} + Ce^{(a+d)/L} = 0$$

- One constant can be expressed as the function of the other:

$$C = -Ae^{-2(a+d)/L}$$

- Substituting this into the general solution gives:

$$\Phi(x) = A\left[e^{-x/L} - e^{x/L-2(a+d)/L}\right]$$

Finite Bare Slab (3)

- The constant A is found from the source condition in the usual way and is:

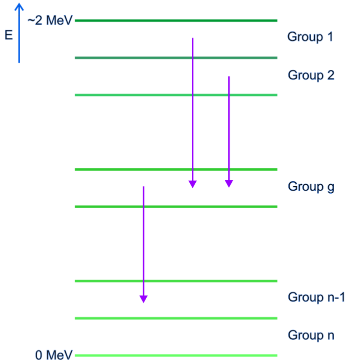
$$A = \frac{SL}{2D}\left(1 + e^{-2(a+d)/L}\right)^{-1}$$

- For positive x-direction, function for Φ is given by:

$$\Phi(x) = \frac{SL}{2D}\frac{e^{-x/L} - e^{x/L-2(a+d)/L}}{1 + e^{-2(a+d)/L}} \tag{6-14}$$

Group Neutron Diffusion

- So far only monoenergetic neutron sources were considered and energy dependence was neglected
- Commonly used form of neutron energy spectrum description is a *group method*



- By this approach, neutrons are divided into energy groups depending on their energy
- If a neutron gains energy, it moves to a higher energy group
- By loosing energy neutron moves into a lower energy group

Group Neutron Diffusion (2)

- Flux in a specific energy range g is calculated as:

$$\Phi_g = \int_g \Phi(E)\,\mathrm{d}E$$

- Neutrons can disappear from a group by absorption and by scattering from one group to another ($g \rightarrow h$):

$$\text{absorption rate} = \Sigma_{ag}\Phi_g,\ \text{group transfer rate} = \Sigma_{g\rightarrow h}\Phi_g$$

- Total transfer rate from one group to another is obtained by summing over all groups that can scatter to the other group
- The group diffusion equation for the gth group can be written in form:

$$D_g\nabla^2\Phi_g - \Sigma_{ag}\Phi_g - \sum_{h=g+1}^{N}\Sigma_{g\rightarrow h}\Phi_g + \sum_{h=1}^{g-1}\Sigma_{h\rightarrow g}\Phi_h = -s_g$$

Two-Group Diffusion Calculation

- For most of the calculations, it is necessary to consider at least two energetic groups
- One group describing thermal neutrons and the other group dealing with neutron moderation (all neutrons above $5kT$)
- The neutron source emits s neutrons of fast neutrons per second
- The source is placed inside infinite uniform diffusion medium

Diffusion Equation for Thermal Energy Group

- With thermal flux defined and possibility to calculate thermal group weighted cross-section, it is possible to formulate one-group time-independent diffusion equation for thermal neutrons:

$$\bar{D}\nabla^2\Phi_T - \bar{\Sigma}_a\Phi_T = -s_T$$, where s_T is thermal neutron source

- Dividing by $\bar{D}$ – thermal diffusion coefficient – gives:

$$\nabla^2\Phi_T - \frac{1}{L_T^2}\Phi_T = -\frac{s_T}{\bar{D}} \tag{6-15}$$

- L$_T$ is thermal diffusion length defined as $L_T^2 = \frac{\bar{D}}{\bar{\Sigma}_a}$

Formulation of Diffusion Equations

- The diffusion equation for the fast group has following shape (42):

$$D_g \nabla^2 \Phi_g - \Sigma_{ag} \Phi_g - \sum_{h=g+1}^{N} \Sigma_{g \to h} \Phi_g + \sum_{h=1}^{g-1} \Sigma_{h \to g} \Phi_h = -s_g$$

- Absorption cross-section for fast neutrons is very small $\rightarrow$ it is possible to neglect the absorption altogether
- We are considering only 2 groups, therefore each neutron scattered in the fast group will reach thermal energy $\rightarrow \Sigma_{1 \to 2} \equiv \Sigma_1$
- Possibility of scattering from the thermal group into the fast group is neglected
- The equation is solved outside the point neutron source and there are no other neutron sources
- The diffusion equation is reduced to the following formulation:

$$D_1 \nabla^2 \Phi_1 - \Sigma_1 \Phi_1 = 0 \tag{6-16}$$

Diffusion Equations Formulation (2)

- Thermal neutron flux is described by equation (6-15)

$$\nabla^2\Phi_T - \frac{1}{L_T^2}\Phi_T = -\frac{s_T}{\overline{D}}$$

- Source of thermal neutrons are neutrons scattered from the fast group into the thermal group $\rightarrow \Sigma_1\Phi_1$
- Diffusion equation for thermal neutrons has the following form:

$$\nabla^2\Phi_T - \frac{1}{L_T^2}\Phi_T = -\frac{\Sigma_1\Phi_1}{\overline{D}} \qquad \text{(6-17)}$$

- For determination of the thermal flux, we must first determine the fast neutron flux

Neutron Age

- We define parameter τ_T – *neutron age*:

$$\tau_T = \frac{D_1}{\Sigma_1} \tag{6-18}$$

- Neutron age gives relation to the time required for slowing-down neutron to the thermal energy
- Unit of τ_T is cm^2
- By introducing neutron age into equation (6-16) we get:

$$\nabla^2\Phi_1 - \frac{1}{\tau_T}\Phi_1 = 0 \tag{6-19}$$

Solution the Diffusion Equation for the Fast Group

- After introducing the shape of Laplace operator for spherical coordinates, the diffusion equation for thermal neutrons has shape identical to equation (6-12)
- Neutron flux of fast neutrons is given by the following formula

$$\Phi_1 = \frac{Se^{-r/\sqrt{\tau_T}}}{4\pi D_1 r}, r \neq 0 \tag{6-20}$$

- The physical meaning of neutron age is similar to the case of diffusion length
- The solution is identical except for the fact that neutrons do not disappear by absorption, but only by scattering into the thermal group

Physical Meaning of Neutron Age

$$\tau_T = \frac{1}{6}\overline{r^2} \tag{6-21}$$

Neutron age gives 1/6 of square of the average distance neutron travels, from the place where it was released to the place where it was slowed-down to the thermal energy

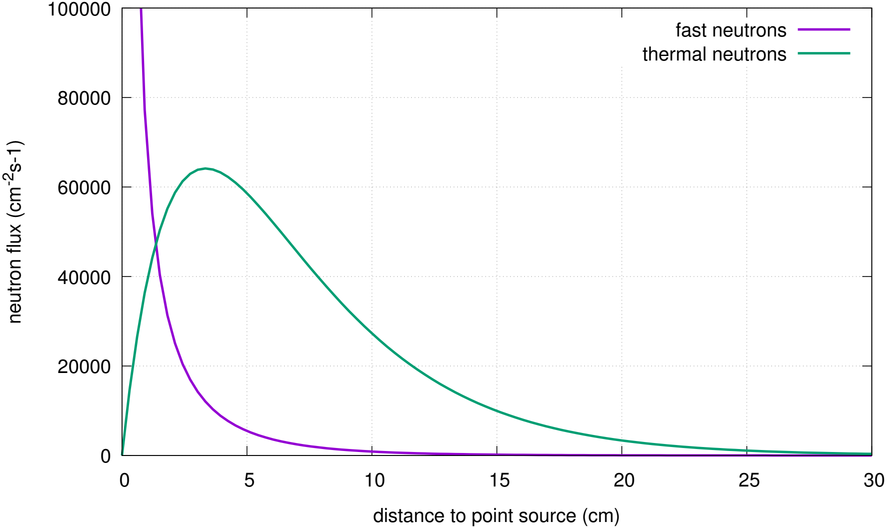
Solution of Diffusion Equation for the Thermal Group

- Introducing the solution for fast neutron flux (6-20) into the diffusion equation for thermal neutrons (6-17) a new equation with nonzero right side is obtained
- The solution is:

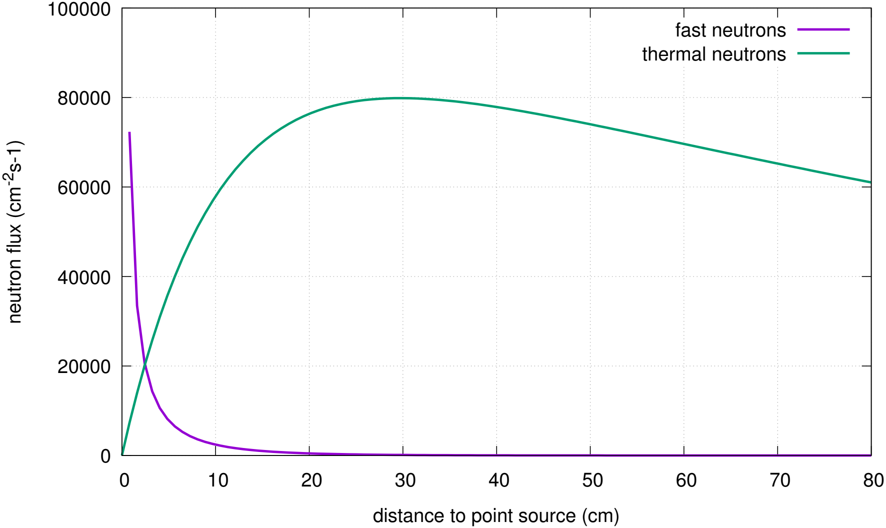
$$\Phi_T = \frac{S\, L_T^2}{4\pi \overline{D}(L_T^2 - \tau_T)} \left(e^{-r/L_T} - e^{-r/\sqrt{\tau_T}} \right) \qquad \text{(6-22)}$$

- This equation gives spatial distribution of thermal neutrons as a result of presence of the source of fast neutrons

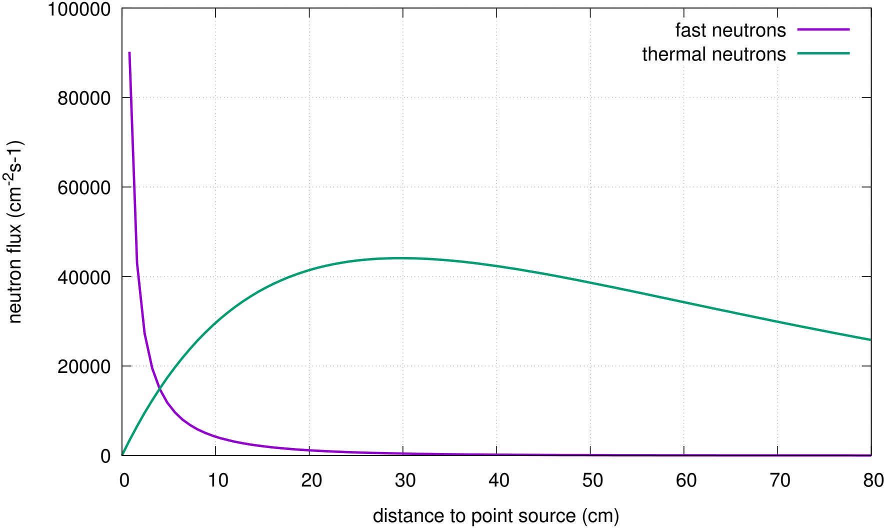
2G Diffusion Solution for H_2O



2G Diffusion Solution for D_2O



2G Diffusion Solution for Graphite



Important constants for moderators for the fast neutron group

Moderator	D_1 (cm)	Σ_1 (cm^{-1})	τ_T (cm^2)
H_2O	0.92	0.0489	19
D_2O	1.26	0.0116	109
Be	0.55	0.0090	61
Graphite	1.04	0.0037	278

Diffusion parameters for moderators in the thermal neutrons group for temperature 20°C

Moderator	density (g/cm^3)	$\overline{D}$ (cm)	L_{TM} (cm)
H_2O	1.00	0.13	2.62
D_2O	1.10	0.76	147
Be	1.85	0.45	22
Graphite	1.60	0.87	61