Neutron Spectra and Thermalization

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Neutron Spectra in Nuclear Reactors

- Neutrons are born with high average kinetic energy (ca 2 MeV)
- They are slowed-down in thermal reactors in order to increase probability of fission
- It is necessary to have a way to approximate neutron spectrum by limited number of simple functions
- The neutron spectrum is divided into three regions:
  - Maxwellian spectrum (1×10^{-5} eV–1 eV)
  - slowing-down spectrum (1 eV–100 keV)
  - fission and potential fusion spectrum (100 keV–20 MeV)

Maxwellian Spectrum

- The Maxwellian spectrum is expressed by the following formula:

$$\chi(E) = C_1 E \exp\left(-\frac{E}{kT}\right) \tag{4-1}$$

- The term kT depends on material temperature, it is equal to 0.0253 eV for pro 20 °C
- This value is usually increased in order to better reflect real systems, it is usually a typical value would be $kT = 0.054$ eV

Slowing-Down and Fission Spectrum

- The slowing-down spectrum follows the Maxwellian spectrum:

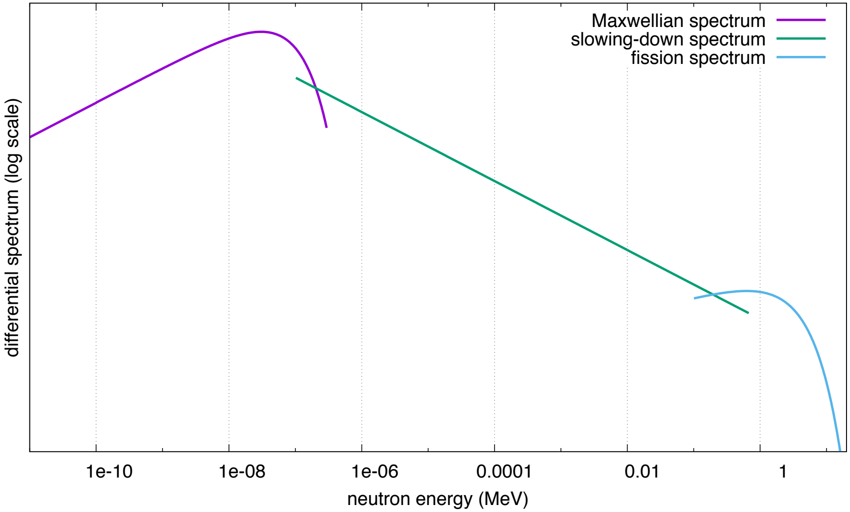
$$\chi(E) = \frac{C_2}{E} \tag{4-2}$$

- Spectrum of fission neutrons can be approximated by formula:

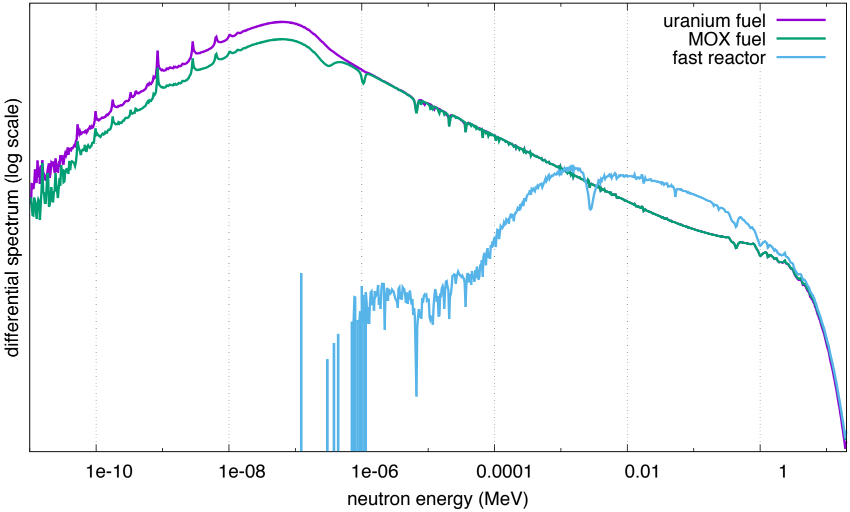
$$\chi(E) = C_3 \sqrt{E} \exp\left(-\frac{E}{\Theta_{\mathrm{fis}}}\right) \tag{4-3}$$

- Parametr $\Theta_{\mathrm{fis}} = 1.4$ MeV
- The spectra can be joined together by constants C_1, C_2, and C_3

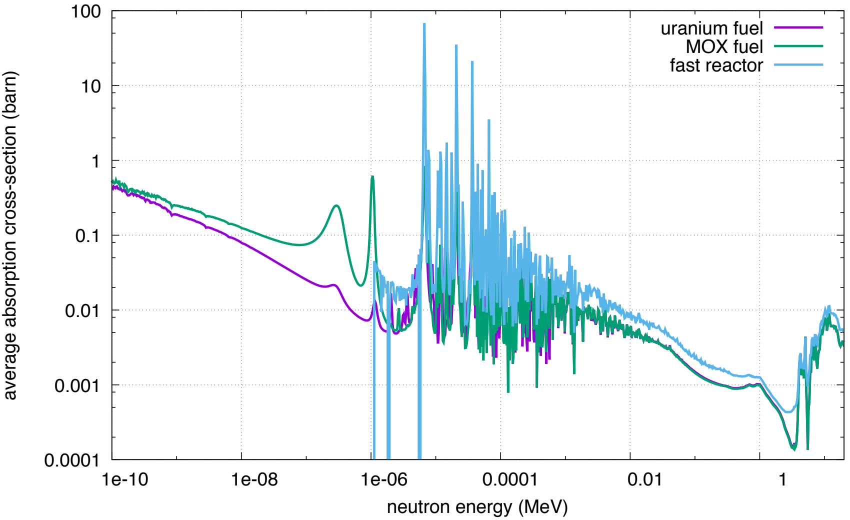
Neutron Spectra Approximation



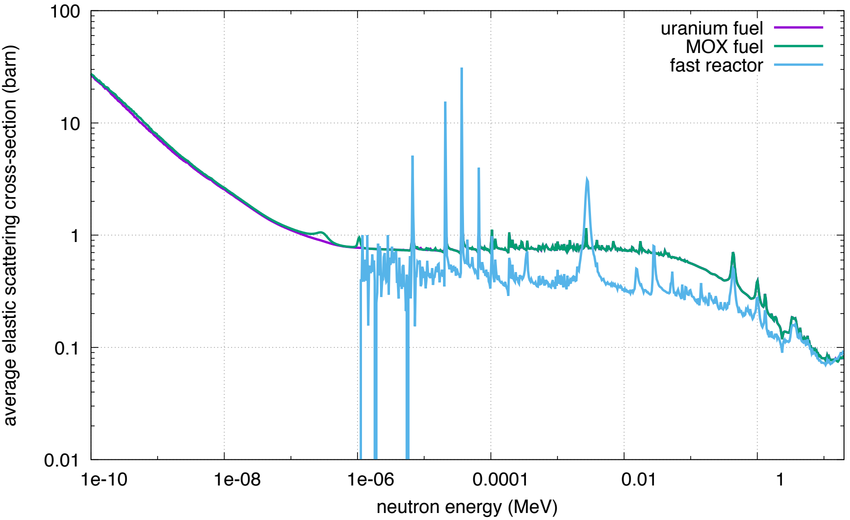
Realistic Neutron Flux Energy Distribution



Microscopic Absorption Cross-Section



Microscopic Elastic Scattering Cross-Section



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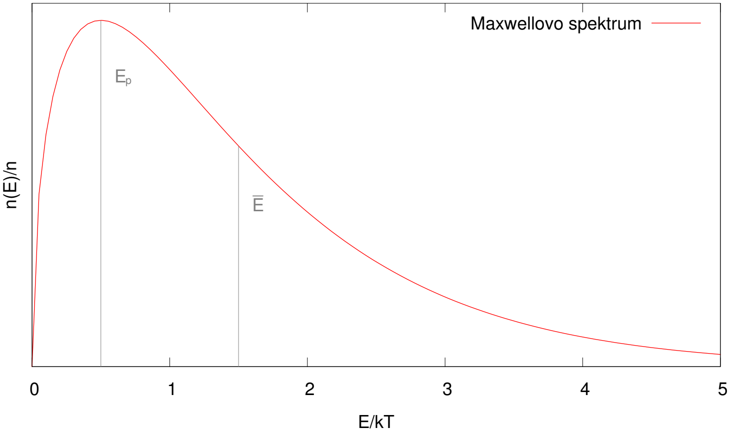
Maxwellian Distribution of Energy

$$n(E) = \frac{2\pi n}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT} \tag{4-4}$$

- $n(E)\,\mathrm{d}E$ is number of particles per unit volume having energies in interval (E, E+dE)
- T is the temperature of the system
- k is Boltzmann's constant, $k = 8.6170 \times 10^{-5}$ eV/K

Maxwellian distribution of energy (cont'd)

- Most probable energy: $E_p = \frac{1}{2}kT$
- Average energy: $\overline{E} = \frac{3}{2}kT$



Maxwellian distribution of velocity

$$n(v) = 4\pi n \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} \tag{4-5}$$

- Most probable velocity: $v_p = \left(\frac{2kT}{m}\right)^{1/2} \rightarrow E(v_p) = kT$
- Average velocity: $\overline{v} = \left(\frac{8kT}{m}\right)^{1/2}$

The value of kT at 20°C is 0.0253 eV.

1/v Cross-Sections

- Absorption cross-section usually depends at low energies at neutron velocity (energy) as 1/v
- The macroscopic cross-section can be thus written as:

$$\Sigma(E) = \Sigma(E_0)\frac{v_0}{v(E)}$$

where E_0 is an arbitrary energy with corresponding speed v_0

$$F = \int n(E)v(E)\Sigma(E)\,\mathrm{d}E = \Sigma(E_0)v_0 \int n(E)\mathrm{d}E = \Sigma(E_0)nv_0$$

For 1/v nuclei the reaction rate is independent of the neutron energy distribution. It is determined by the cross-section at arbitrary energy.

Reaction rate of neutrons with energy distribution is the same as for monoenergetic neutron with energy E_0 and intensity nv_0

Thermal Cross-Section

- Is is convenient to tabulate cross-section data for one selected energy
- It is *thermal neutron energy* $E_0 = 0.0253$ eV with corresponding speed $v_0 = 2200$ m/sec
- Neutron flux at thermal energy is defined as:

$$\Phi_0 = n v_0$$

- And reaction rate for this energy is:

$$F = \Sigma(E_0)\Phi_0$$

Non-1/v Cross-Sections

- For a few important nuclei 1/v law is not valid
- To evaluate reaction rate it is necessary to calculate integral

$$F = \int n(E) v(E) \Sigma(E) \, \mathrm{d}E$$

- Based on the assumption of the Maxwellian distribution of n(E) reaction rate for these important nuclei could be calculated as

$$F = g(T) \Sigma(E_0) \Phi_0$$

where $g(T)$ is *non-1/v factor*

Group Neutron Flux

- Neutrons can be divided into groups based on their energy
- Group neutron flux is defined by equation (4-6):

$$\phi_{\mathrm{g}} = \int_{\mathrm{g}} \phi(E)\, \mathrm{d}E \tag{4-6}$$

- There is special case of thermal neutron flux, where energy distribution of neutron density is known, see (4-4)
- It is possible to calculate thermal neutron flux using the Maxwellian distribution

Thermal Neutron Flux

- Let's calculate thermal neutron flux taking advantage of limited range of the Maxwellian distribution:

$$\phi_\mathrm{T} = \int_\mathrm{T} \phi(E)\,\mathrm{d}E = \int_0^\infty \phi(E)\,\mathrm{d}E = \int_0^\infty n(E)v(E)\,\mathrm{d}E \tag{4-7}$$

$$\phi_\mathrm{T} = \int_0^\infty n(E)\sqrt{\frac{2E}{m}}\,\mathrm{d}E = \frac{2^{3/2}\pi n}{(\pi kT)^{3/2}m^{1/2}}\int_0^\infty E\mathrm{e}^{-E/kT}\,\mathrm{d}E \tag{4-8}$$

$$\phi_\mathrm{T} = \frac{2}{\sqrt{\pi}}nv_\mathrm{T} \tag{4-9}$$

- It is than possible to calculate macroscopic absorption cross-section averaged by the thermal spectrum

Thermal Absorption Cross-Section

- There is general formula for group macroscopic absorption cross-section:

$$\Sigma_{ag} = \frac{\int_g \Sigma_a(E)\phi(E)\, dE}{\int_g \phi(E)\, dE} \tag{4-10}$$

- Thermal absorption cross-section is calculated using thermal neutron flux (4-9):

$$\overline{\Sigma}_a = \frac{\sqrt{\pi}}{2} g_a(T)\Sigma_a(E_0)\left(\frac{T_0}{T}\right)^{1/2} \tag{4-11}$$

Time of Thermal Diffusion

- Thermal reactor are expected
- Neutrons can travel various distances before slowing-down to thermal energy and during following diffusion depending on the moderator type
- There are also various lifetimes of prompt fission neutrons depending on the moderator type
- Lifetime of a neutron is divided into slowing-down time, time of thermalization, and time of thermal diffusion
- It is assumed that only loss of kinetic energy is possible during elastic collisions of neutrons with kinetic energy above 1 eV - Slowing-down region
- There is thermalization range between 5 kT and 1 eV, up-scattering is possible
- The thermal neutron energy distribution below 5 kT corresponds to thermal motion of the moderator

Slowing-Down and Thermal Diffusion Time

Moderator	H_2O	D_2O	Be	BeO	graphite
t_m (µs)	1.0	8.1	9.3	12	23
t_{th} (µs)	5	66	45	72	200
t_d (µs)	210	140000	3900	6700	17000

Thermal Diffusion Time

- The thermal diffusion time is the dominant component of prompt neutron lifetime
- It can be expressed as the average time neutron spends in an infinite system before absorption

$$t(E) = \frac{\lambda_a(E)}{v(E)} = \frac{1}{\Sigma_a(E)v(E)} \overset{1/v}{=} \frac{1}{\Sigma_a(E_0)v_0}$$

$$t_\mathrm{d} = \frac{1}{\Sigma_a(E_0)v_0} = \frac{\sqrt{\pi}}{2\overline{\Sigma}_a v_T} \qquad (4\text{-}12)$$