Neutron Slowing-Down

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Center-of-mass system

- It is usually expected that the target nucleus is at rest in the laboratory system
- This is never true for center-of-mass system, where the center-of-mass is at rest and both neutron and target nucleus are moving to this center
- Velocity of the center-of-mass $\boldsymbol{v_0}$ can be calculated as follows:

$$\boldsymbol{v_0} = \frac{m\,\boldsymbol{v_L} + M\,\boldsymbol{V_L}}{m + M} \tag{3-1}$$

- There are also following relations between velocity in laboratory and center-of-mass system:

$$\boldsymbol{v_C} = \boldsymbol{v_L} - \boldsymbol{v_0} \tag{3-2}$$

Velocity in center-of-mass system

- It is possible to derive approximate formulas for velocity of the center-of-mass and neutron in the center-of-mass using equations (3-1) and (3-2)
- It is assumed that mass of target nucleus divided by neutron mass equals approximately mass number A
- Center-of-mass speed:

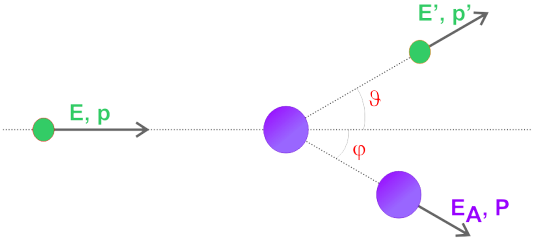
$$\boldsymbol{v_0} \approx \frac{1}{1+A}\boldsymbol{v_L} \tag{3-3}$$

- Neutron velocity:

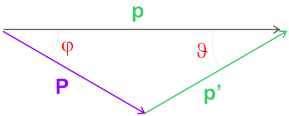
$$\boldsymbol{v_C} = \boldsymbol{v_L} - \frac{m}{m+M}\boldsymbol{v_L} = \frac{M}{m+M}\boldsymbol{v_L} \approx \frac{A}{1+A}\boldsymbol{v_L} \tag{3-4}$$

Energy Loss in Elastic Scattering Collisions

- Neutron is elastically scattered by a nucleus at rest
- Kinetic energy and momentum must be preserved
- E, **p** and E', **p'** are kinetic energy and momentum of the neutron before and after collision, respectively
- E_A, **P** are kinetic energy and momentum of the nucleus after collision



Energy Loss Calculation



- Conservation of energy $E = E' + E_A$ and momentum $\mathbf{p} = \mathbf{p'} + \mathbf{P}$

$$P^2 = p^2 + p'^2 - 2pp' \cos\vartheta \text{ (dot product of vectors)}$$

- Momentum is calculated as: $p = mv = m\sqrt{2E/m} = \sqrt{2mE}$
- M is mass of the nucleus, m is mass of the neutron
- M/m equals approximately A (atomic mass number) of the nucleus

$$ME_A = mE + mE' - 2m\sqrt{EE'}\cos\vartheta$$

- Introducing $E_A = E - E'$ and A gives:

$$A(E - E') = E + E' - 2\sqrt{EE'}\cos\vartheta$$

Energy Loss Calculation (cont'd)

- Previous equation rearranged:

$$(A+1)E' - 2\sqrt{EE'}\cos\vartheta - (A-1)E = 0$$

- This equation is quadratic in $\sqrt{E'}$ and has solution:

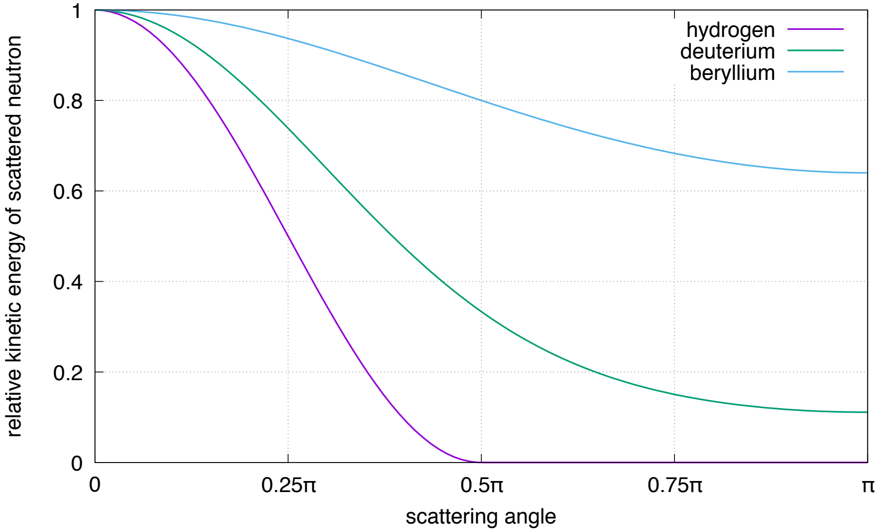
$$E' = \frac{E}{(A+1)^2}\left[\cos\vartheta + \sqrt{A^2 - \sin^2\vartheta}\right]^2 \qquad (3\text{-}5)$$

- Grazing collision – ϑ is zero $\rightarrow$ E'=E
- Value of E' is minimum when $\vartheta = \pi$ – backward scattering

$$(E')_{min} = \left(\frac{A-1}{A+1}\right)^2 E = \alpha E \qquad (3\text{-}6)$$

where $\alpha = \left(\frac{A-1}{A+1}\right)^2$ is *collision parameter*

Energy loss


hydrogen
deuterium
beryllium
relative kinetic energy of scattered neutron
1
0.8
0.6
0.4
0.2
0
0
0.25π
0.5π
0.75π
π
scattering angle

Energy of scattered neutron

- It is possible to calculate velocity of a scattered neutron in the laboratory system using its velocity in the center-of-mass system
- Velocity of the scattered neutron in the center-of-mass system does not change

$$(v_L')^2 = (v_0)^2 + (v_C)^2 + 2 v_0 v_C \cos\Theta$$

$$\begin{aligned}(v_L')^2 &= \left(\frac{v_L}{A+1}\right)^2 + \left(\frac{A v_L}{A+1}\right)^2 + \frac{2 A v_L^2}{(A+1)^2}\cos\Theta \\ &= \frac{v_L^2 (A^2 + 2A\cos\Theta + 1)}{(A+1)^2}\end{aligned}$$

- This equation is also valid for kinetic energy in the laboratory system

Energy of scattered neutron (cont'd)

- The above equation can be simplified by introducing the collision parameter:

$$E_L' = \frac{1}{2} E_L \left[(1+\alpha) + (1-\alpha)\cos\Theta\right] \qquad (3\text{-}7)$$

- The minimum energy of the scattered neutron again depends on the collision parameter
- It can be shown that average energy of elastically scattered neutron by light elements is given approximately by:

$$\overline{E'} = \frac{1}{2}(1+\alpha)E$$

- Average fractional energy loss:

$$\frac{\overline{\Delta E}}{E} = \frac{1}{2}(1-\alpha)$$

Average Loss of Energy (cont'd)

- Through elastic collisions with light nuclei neutron looses more energy than by collisions with heavy nuclei
  - hydrogen: $\alpha = 0$, $\overline{\Delta E}/E = 50\%$
  - carbon: $\alpha = 0.716$, $\overline{\Delta E}/E = 14\%$
  - uranium: $\alpha = 0.983$, $\overline{\Delta E}/E = 1\%$
- Equation for average fractional energy loss is not valid for high-energy neutrons. At higher energies the energy loss in collision is less than predicted by the formula.

Neutron Lethargy

- Neutron lethargy (u) is defined by formula:

$$u = \ln\left(\frac{E_M}{E}\right) = \ln(E_M) - \ln(E) \tag{3-8}$$

  - E_M is arbitrary energy, usually the highest neutron energy in the system
  - E neutron energy

There is low lethargy for high-energy neutrons. Lethargy increases as energy of a neutron is decreasing.

Average Change in Lethargy

- Average change in lethargy $\overline{\Delta u}$ is independent of the energy of the incident neutron
- The quantity $\overline{\Delta u}$ is often used and is denoted as ξ
- It can be shown that:

$$\xi = 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right)$$

- There is also simplified formula:

$$\xi = \frac{2}{A + \frac{2}{3}} \tag{3-9}$$

- This simple formula is valid for heavy nuclei. The difference from correct result for A = 2 is about 3 %

Neutron Slowing-Down and Diffusion

- Neutrons are produced by fission at high energies and slowed-down in order to increase probability of initiating new fission
- Effectiveness of energy decrease depends on the character of target nucleus
- Slowing down neutrons in a few collisions reduces leakage from the core and resonance absorption (it will be discussed later)
- When a neutron reaches energy lower than 1 eV, elastic scattering reactions are the most important – *diffusion*

Properties of an ideal moderating material

- Large scattering cross-section
- Small absorption cross-section
- Large energy loss per collision

Number of Collisions to Thermalization

- ξ is the average logarithmic energy loss
- Number of collision required to slowing down from higher energy to lower energy is:

$$N = \frac{\ln(E_{high}) - \ln(E_{low})}{\xi} = \frac{\ln\left(\frac{E_{high}}{E_{low}}\right)}{\xi}$$

For example: Slowing down a neutron with an average energy from fission energy 2 MeV to the thermal energy 0.0253 eV

- by H_2O – 19 collisions
- by D_2O – 35 collisions
- by Be – 86 collisions
- by C – 114 collisions
- by Fe – 510 collisions
- by U – 2168 collisions

Macroscopic Slowing Down Power

- This value characterises the moderating capability of all the nuclei in 1 cm^3
- Indicates how rapidly a neutron will slow down in the material
- With increasing value of MSDP increases also the capability of slowing down of neutrons

$$MSDP = \xi \Sigma_s \tag{3-10}$$

Moderating Ratio

- Characterises the overall effectiveness of a moderating material taking into account not only scattering reactions, but also possible neutron absorption
- Even moderators have possibility to absorb neutrons during slowing down
- Effective moderator has low neutron absorption cross-section
- Higher MR means higher effectiveness of the moderating material

$$MR = \frac{\xi \Sigma_s}{\Sigma_a} \tag{3-11}$$

Moderating Properties of Selected Light Materials

Material	ξ	NCT*	MSDP [cm^{-1}]	MR
H_2O	0.927	19	1.425	62
D_2O	0.510	35	0.177	4830
Be	0.207	86	0.154	126
B	0.171	105	0.092	0.00086
C	0.158	114	0.083	216

*Number of collisions required to reach thermal energy